Introduction

Short Description

This programme is intended to enrich the mathematical education in secondary schools.

Not only teachers will benefit from this programme in their preparations of lessons but also students will profit by exploring, testing, constructing, calculating and visualizing geometrical objects.

This version of the programme contains many elements of an „**interactive geometry**“. In many cases objects can be moved or resized with the mouse. Depending objects, like images of a transformation, are adapted during the movement.

All objects are shown in a 2d-coordinate-system with numerous options either with equidistant or individual scales.

The offered geometrical objects are the following:

Point, line segment, vector, half-line, straight line, angle, triangle, rectangle, polygon, circular sector, circle, sector of an ellipse, ellipse.

These objects can be changed by various options and they can be moved with the mouse (if not achored). Especially with triangles you can find even more options: It is possible to draw mid-perpendicular, bisectrix of angle, median and height as well as incircle and circumcircle. In addition squares over all sides can be drawn and there are special options for rectangular triangles. For example, a shear animation can be started, visualizing the proof of the theorem of Pythagoras.

Also the options of ellipses have been enhanced. Now the focus points and the focal radii can be shown and changes of semi-axes, rotation and translation with the mouse have been implemented.

You can input tables of values and calculate approximations by functions.

New to version 7 is the option to generate a Bézier curve along control-points (with or without weights). Moving the control-points will adapt the curve automatically.

You can calculate distances and intersections for many combinations of objects.

The most important geometrical objects can be mapped by affine transformations:

$$\vec{x}' = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \vec{x} + \begin{pmatrix} e \\ f \end{pmatrix}$$

Here too, the principle of interactive geometry has been put into practice. If, for example, a point acting as centre of scaling is moved, all corresponding images are adapted during the movement.

Additionally the programme offers the opportunity of drawing functional graphs. This works with functions of one variable and with parametric functions (x(t);y(t)). You can find several options like the calculation of zeroes, extremes and integrals. Also the intersection of functional graphs can be calculated.

Furthermore you can draw the Mandelbrot set and Julia sets. Interesting regions of these sets can be zoomed in by selecting an appropriate rectangle.

The results of calculations are usually shown in the resizable text window. This contents can be saved together with the graphical data or sent to a printer.

It is also possible to save the graphical scene as an enhanced metafile (*.**emf**), in postscript format (***.eps**) or as a bitmap (*.**bmp, *.jpg, *.png**). This way you can import the saved files into word processing software to produce worksheets or educational material.

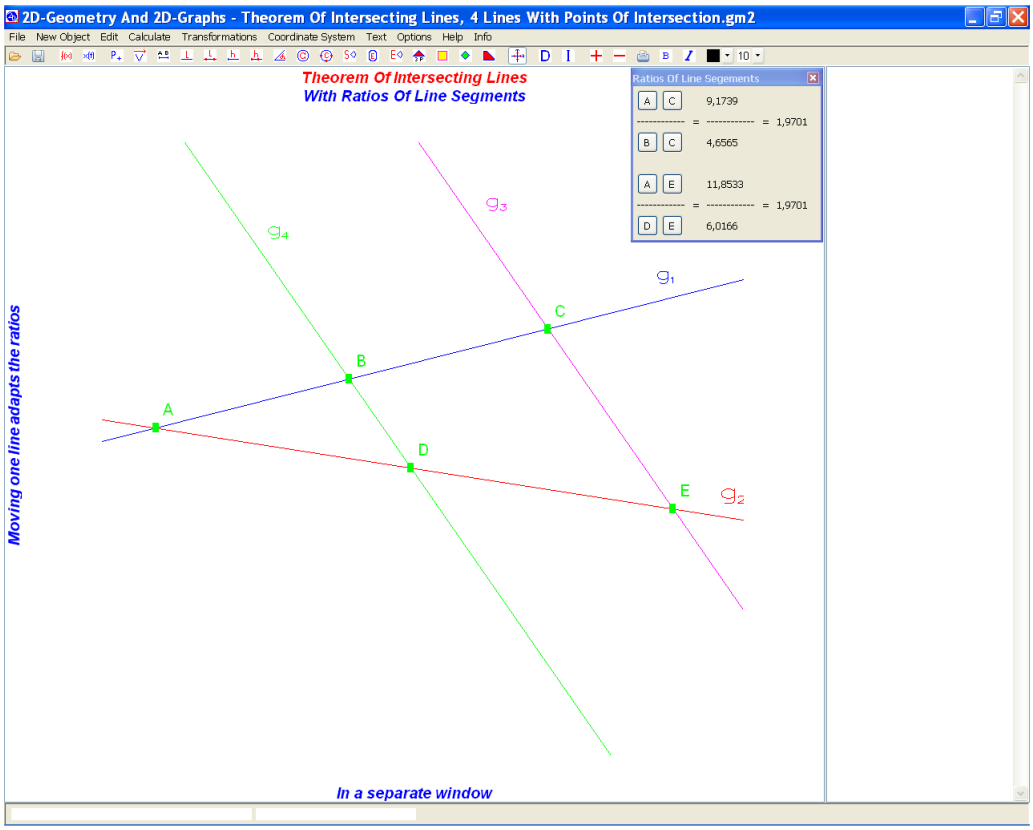
A big emphasis has been laid on a fluent operability. Accessing the options of an object by right clicking on it is one example of this.

The Screen

Graphical Window

Beside the main menu and the toolbar there is a sizable data window in the right quarter.

The main region is used for drawing the objects. On top of this region there are two title areas, below there is one as well as on the left.



The data window is visible all the time. As default a previous output in the data window will be overwritten by the new output, so that only the actual data can ve viewed. This setting, however, can be changed by right clicking in the data window and selecting insert mode. This way it is possible to produce a text file containing all the data of the objects you selected for output. It is possible to save these data as a text file.

You can add text of your own in the data window and the text will scroll if necessary.

Attention: The data window is a pure text window. Changes you make to the data in this window will not change the corresponding object data in the drawing, which originally produced the output.

The Main Menu

Overview

These are the items of the main menu:

- File
- New Object
- Calculation
- Transformation
- Coordinate System
- Text
- Options
- Help
- Info

Menu-File

The dropdown menu contains the following items:

- New
- Open
- Save
- Save As
- Export Graphic
- Print Page Settings
- Print

File – New

The actual drawing will be closed and you start with a coordinate system without further objects.

File – Open

Previously saved drawings can be reopened. This programme uses a special file format with the extension *.gm2.

file – Save

The actual drawing can be saved including all titles and labels. The format used for saving is a vector format. Opening the saved file (with this programme) on a computer with higher resolution graphics will show the objects with the better resolution.

File - Save As

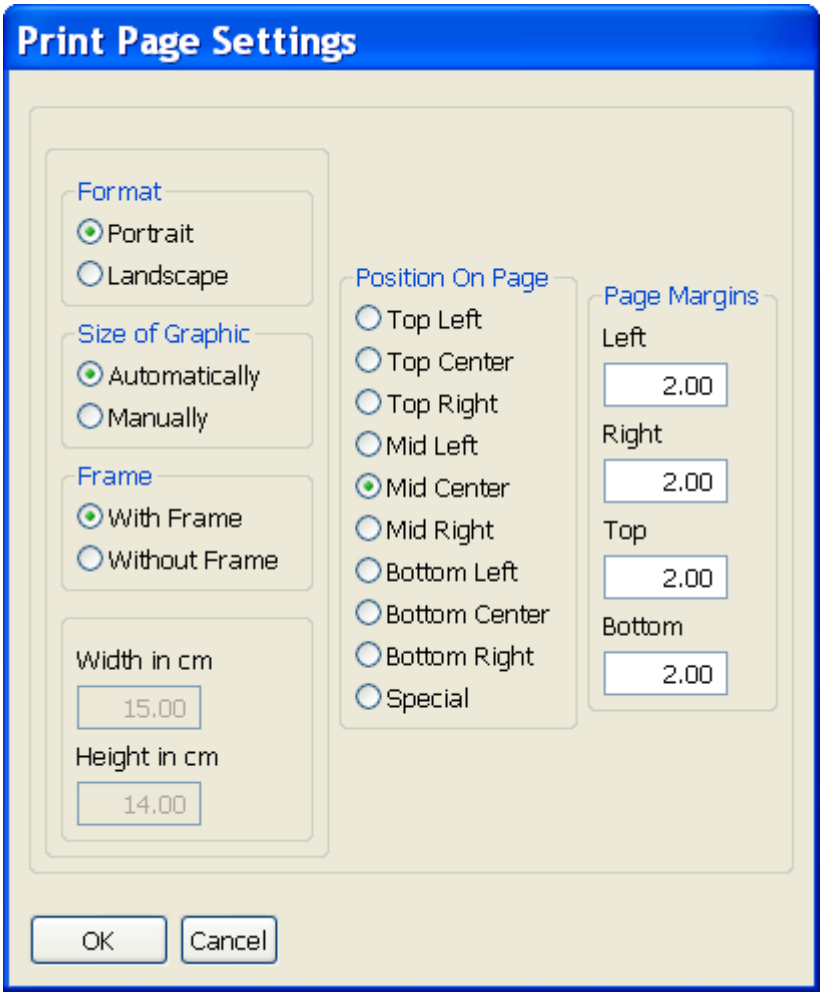
This action saves the drawing with a new filename.

File – Export Graphic

Choosing this item leads to a submenu where three choices are available: Enhanced Metafile (*.emf), pixel formats and postscript format (*.eps). As pixel formats you can use *.png, *.bmp, *.jpg or *.tif. You select one of these items, if you intend to import the drawing - arbitrarily scaled - into a word processing programme.

File – Print Page Settings

The print page settings dialog looks like this:



The single items are self explanatory. However, you have to keep in mind that selecting „Print In CM-Units“ may change the printed section of the coordinate system significantly!

File – Print

This item will print the drawing according to the print page settings.

Menu New Object

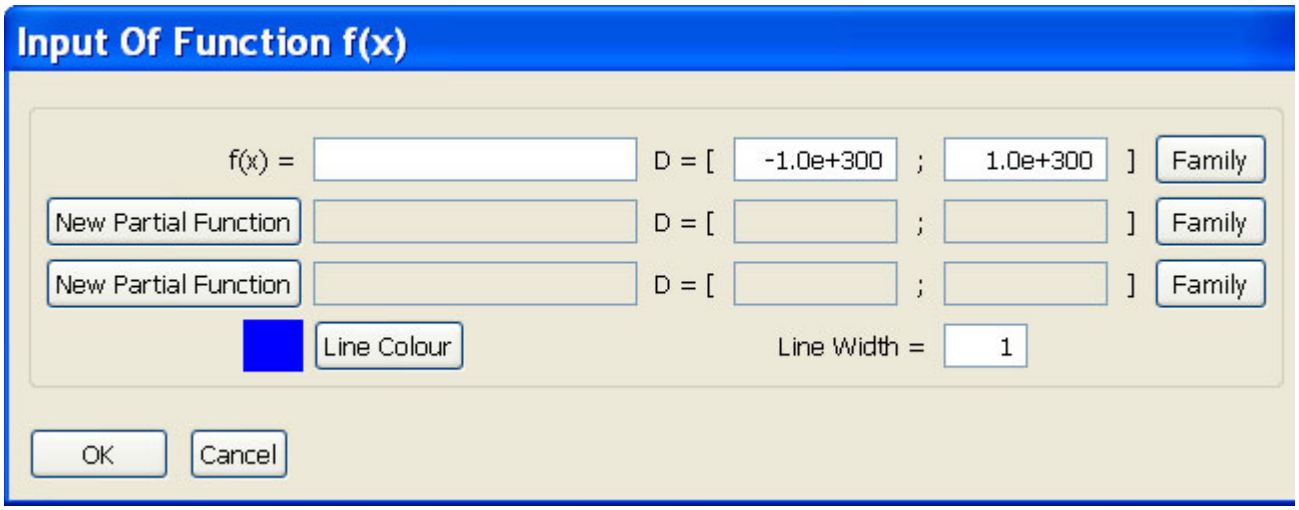
Objects created using this menu are anchored by default. This means that they cannot be moved without explicitly allowing it. This can be done via the object's local menu (right clicking on the object).

- Functios
- Point
- Vector
- Line Segment
- Halfline
- Straight Line
- Angle
- Triangle
- Rectangle
- Polygon
- Circle
- Circular Section
- Ellipse
- Sector Of An Ellipse
- Table Of Measurement
- Bézier Curve
- Mandelbrot Set
- Julia Set
- Ratio Of Line Segements

New Object – Function

You can choose between a function of one variable with symbol f(x) and a function in parametric form with symbol (x(t);y(t)).

a) f(x)



You can specify whether to define a globally or piecewise defined function. Additionally a family of functions can be generated by using the parameter s in the functional term. There are some rules to follow for these terms. The following table shows the implemented functions and their abbreviations:

Abbr.	Function
arcsin	Arc Sine
arccos	Arc Cosine
arctan	Arc Tangent
sin	Sine Function
cos	Cosine Function
tan	Tangent Function
sinh	Hyperbolic Sine
cosh	Hyperbolic Cosine
ln	Natural Logarithm
log	Decimal Logarithm
int	Integer Part
frac	Decimal Part
exp	Exponential Function
sqrt	Square Root Function
sqr	Square Function

The functional argument has to be placed in round brackets.

An example:

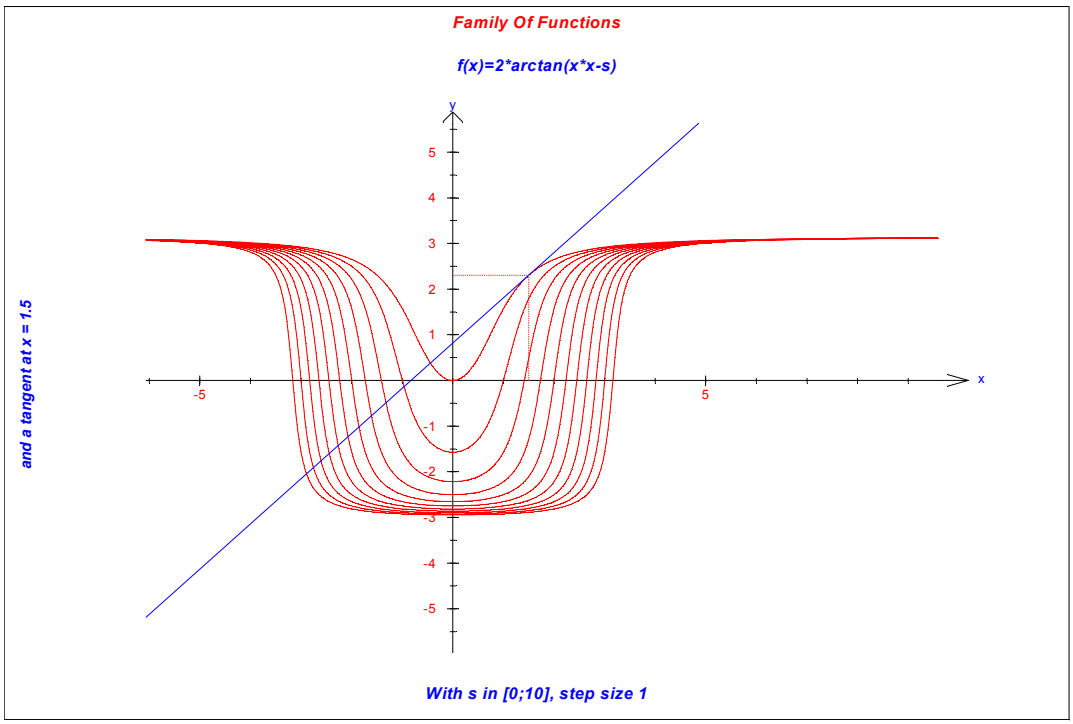
$$f(x) = 2 \cdot e^{-\frac{1}{2} \cdot x^2}$$

Tthe corresponding input:

2*exp(-1/2*x^2)

Pushing the „Family“-button opens a dialogue with details.

A sample output:

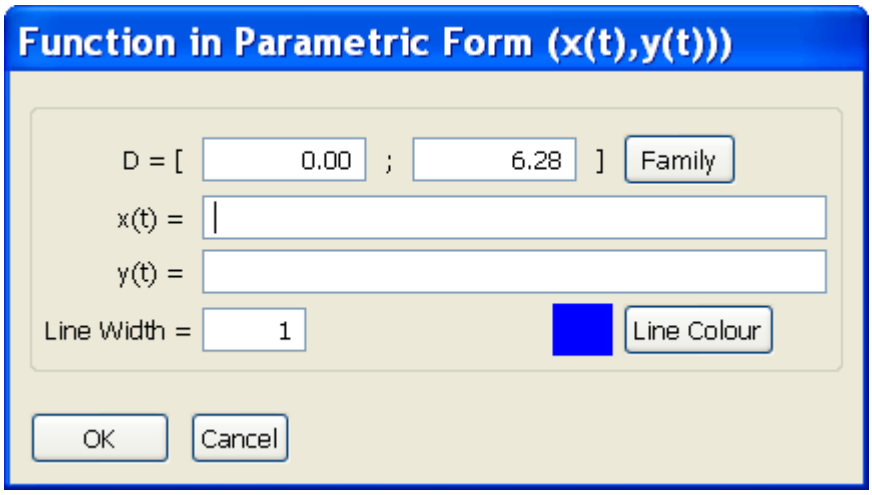


b) (x(t);y(t))

You can define functional terms for x(t) and y(t), where you have to follow the same rules as indicated above.

This option should be of interest to physicists because it offers the opportunity of drawing paths of particles.

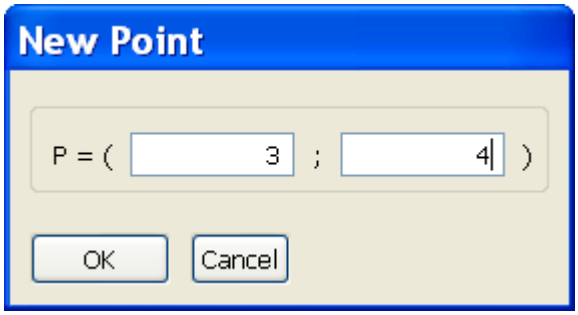
Here, too, families of functions can be defined by using the parameter s.



New Object – Common Features

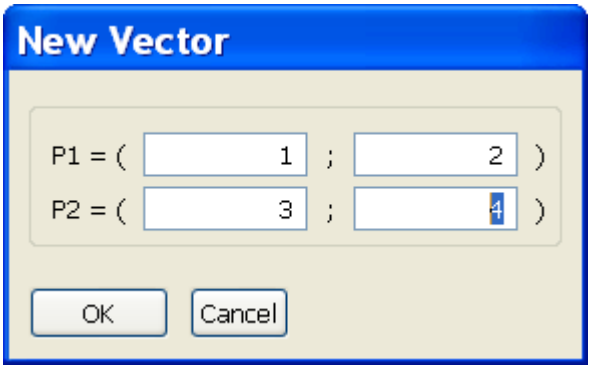
Most of the dialogues are self-explanatory. The required data can be entered with the help of describing labels. Some of the objects show several methods for creation. For example straight lines can be defined either by specifying 2 points or one reference point and a directional vector or the standard line equation.

New Object - Point



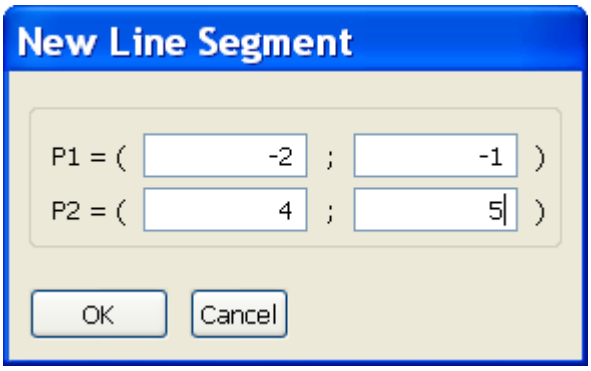
The point coordinates have to be specified.

New Object - Vector



First method: You define the components of the vector dx and dy
Second mehtod: You define the initial point and the end-point.

New Object - Line Segment



You specify initial and end-point.

New Object - Halfline

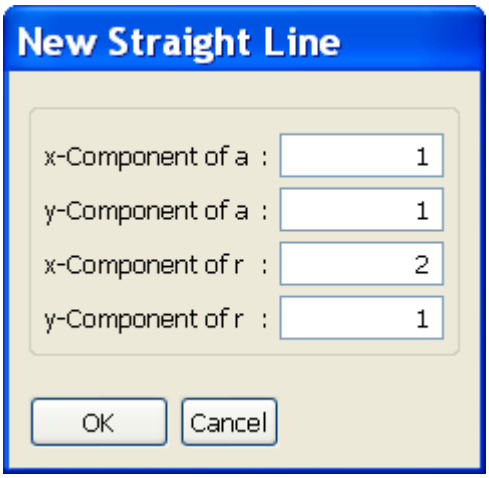
You specify the initial point and an arbitrary point on the halfline.

New Object - Straight Line

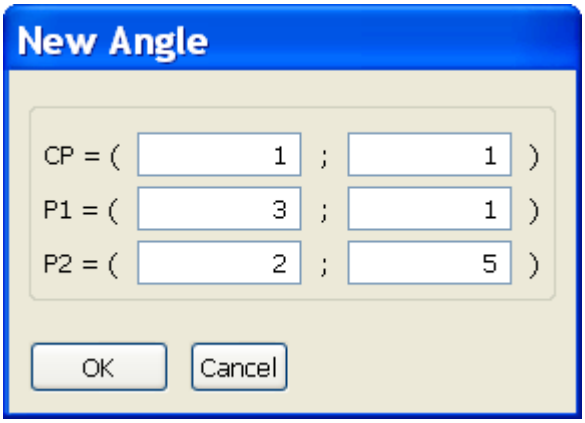
You select one of the following methods:
- you specify two points
- you define the straight line by using the parametric form
- you specify the coefficients of the standard line equation $y = a \cdot x + b$ .. where a is the slope anb b is the section of the y-axis.

Additionally a straight line can be generated by clicking on two existing points. A line created by this method is tied to the points. Moving such a point will change the straight line accordingly.

A sample input dialogue for a straight line in parametric form:

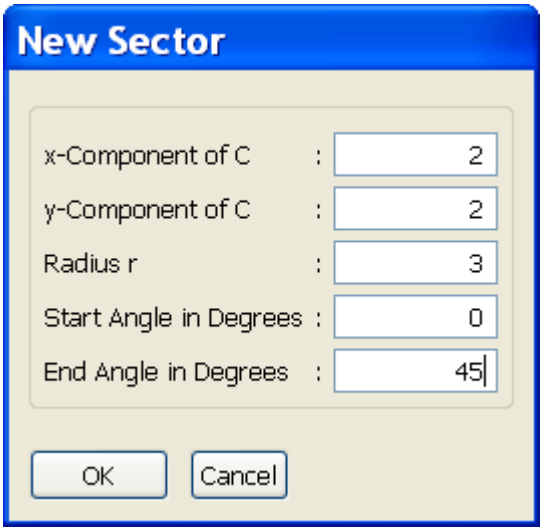


New Object - Angle



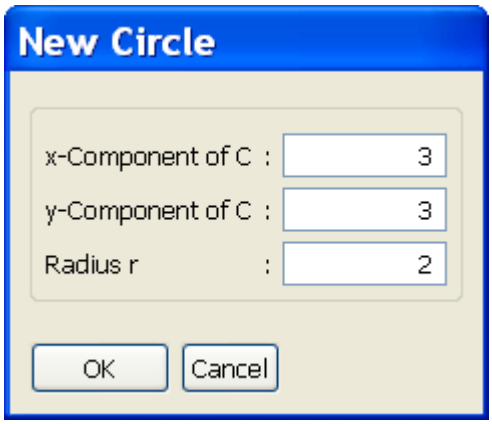
You specify the vertex and two points on the legs.

New Object - Circular Section



Apart from centre and radius you specify initial angle and end-angle, both relative to the positive part of the x-axis (and counterclockwise).

New Object - Circle



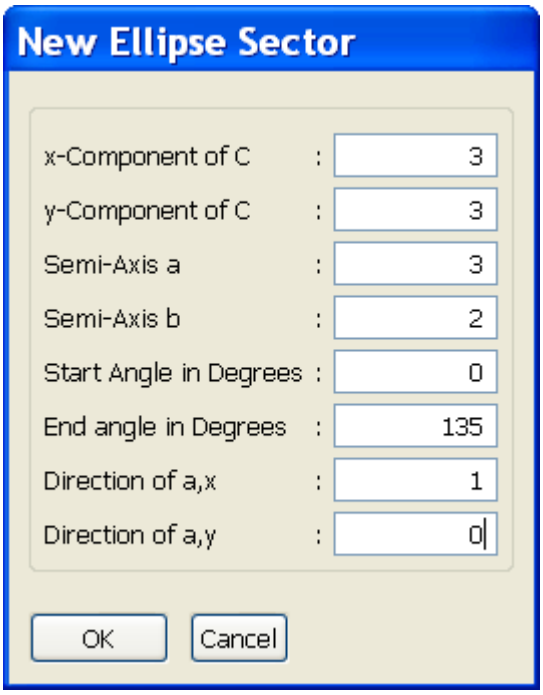
1. method: Centre and radius are required.

2. method: Mouse clicking on three existing points

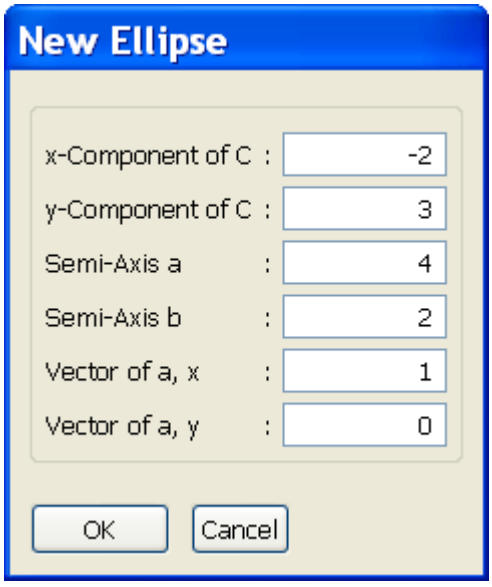
Using the second method sets the programme into selection mode, which leads to an uparrow cursor. You are expected to select three existing points The circle will then be tied to the points. Moving one of the points will adapt the circle.

New Object - Sector Of An Ellipse

Centre, initial and end-angle are required.



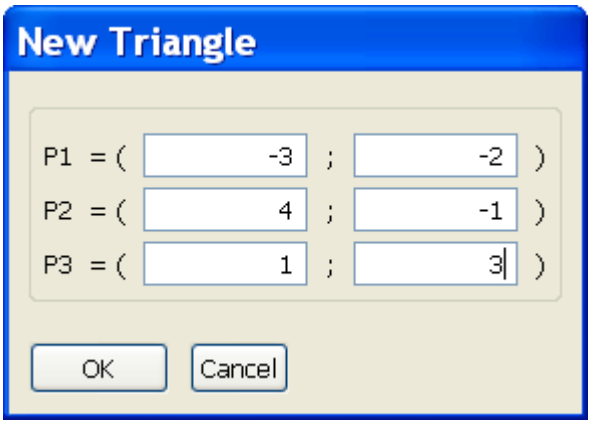
New Object - Ellipse



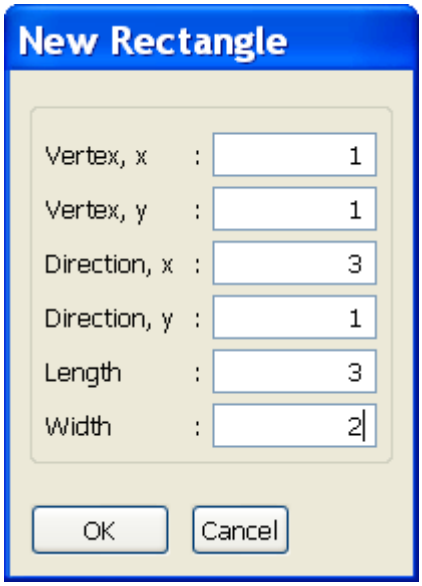
Beside the centre the length of the semi-axes and a direction vector of the semi-axis a have to be determined.

New Object - Triangle

It is possible to specify three points or to work with sentences of congruence and select a set of line segments and angles.



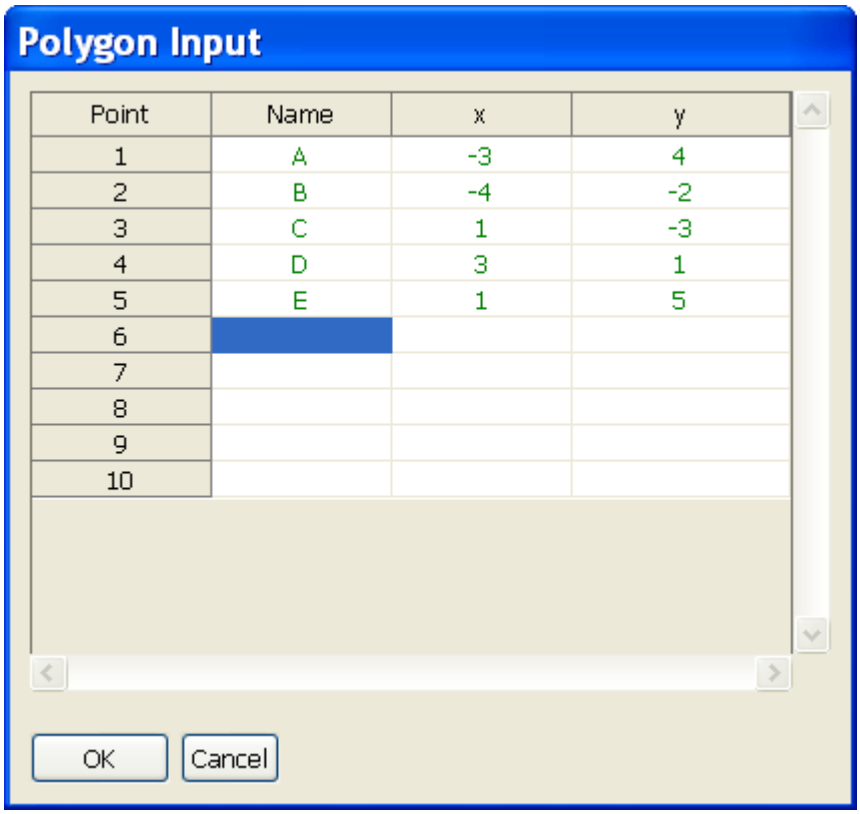
New Object - Rectangle



You are prompted to determine a vertex, the direction vector of one side as well as width and length.

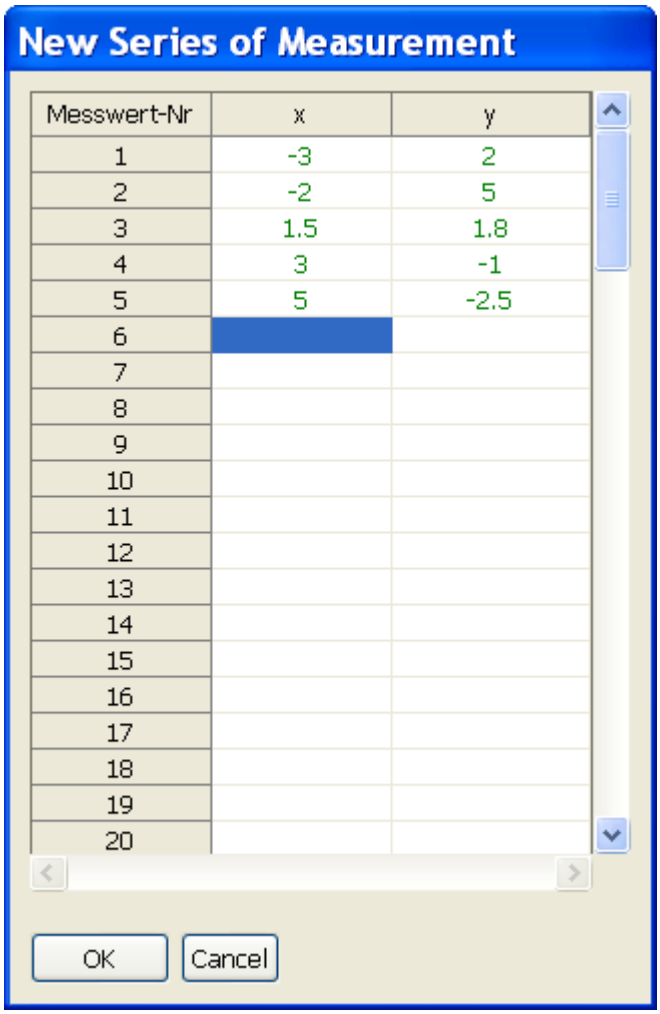
New Object - Polygon

You can select between an arbitrary polygon (4 to 10 vertices) and a regular one.



New Object – Table Of Measurement

This dialogue offers a table where you can specify the x- and y-coordinates of the points of measurement.



Clicking the OK-button leads to a data check. If the content of a field cannot be interpreted as a number, this cell gets the focus to allow for correction.

This is also true for uncomplete pairs of components.

If no errors are detected, all the points are saved and drawn.

Right clicking on a data point opens the local dropdown menu, which offers several options for the representation of the data points.

New Object - Bézier

You can fill out a table fo points. These points are the control-points of a Bézier curve, that represents a continuous approximation through the points. Weights can be specified.

Menu Edit

- Back to state before deletion

This option restores the state that was present before the last delete operation.

Menu Calculation

- Distance
- Intersection
- Integral between ...
- Angle measurement by selecting 3 points

Calculation – Distance or intersection

Choosing this item turns the programme into **selection mode**. The cursor turns into an up-arrow. Now you have to click on the objects you want. The programme then calculates the distance or the intersection. The correcpon-ding object (point or list of points) will be added to the appropriate list of objects.

Integral between ...

This option relates to functions of one variable and expects you to select the two desired functions. The programme will then calculate the points of inter-section and the area between the curves in the calculated interval.

Angle measurement by selecting 3 points

The programme expects you to first select the angular point. The two follo-wing point-clicks are interpreted as belonging to the first and second leg of the angle (counterclockwise).

This operation does not create an angle object!

Menu Transformation

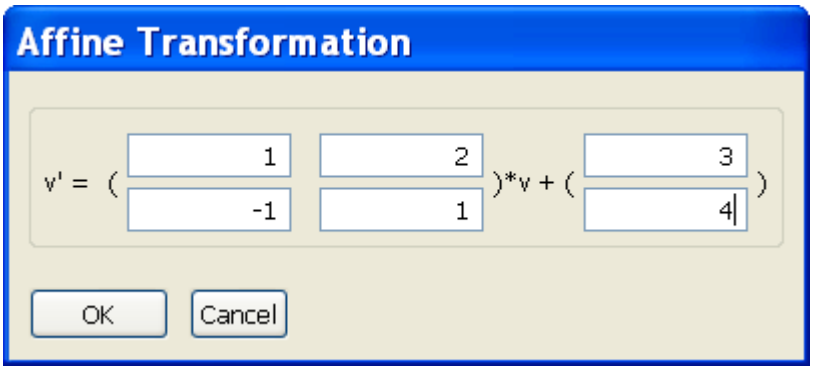
The programme offers affine transformations of most of the objects in the scene. For this purpose a number of input dialogues is available:

You can choose between:

- Rotation
- Translation
- Reflection with respect to a straight line
- Reflection with respect to a point
- Shear
- Scaling with respect to a point
- General affine transformation

There are two ways to specify the necessary data. The first is through the main menu, the second is through the local menus of the objects. For example, to define a reflection with respect to a straight line, that is already in the scene, you right-click on the line and choose the item **Select As ...** . In the following dropdown menu you choose **Line Of Reflection**.

This is a sample input dialogue of a general affine transformation:



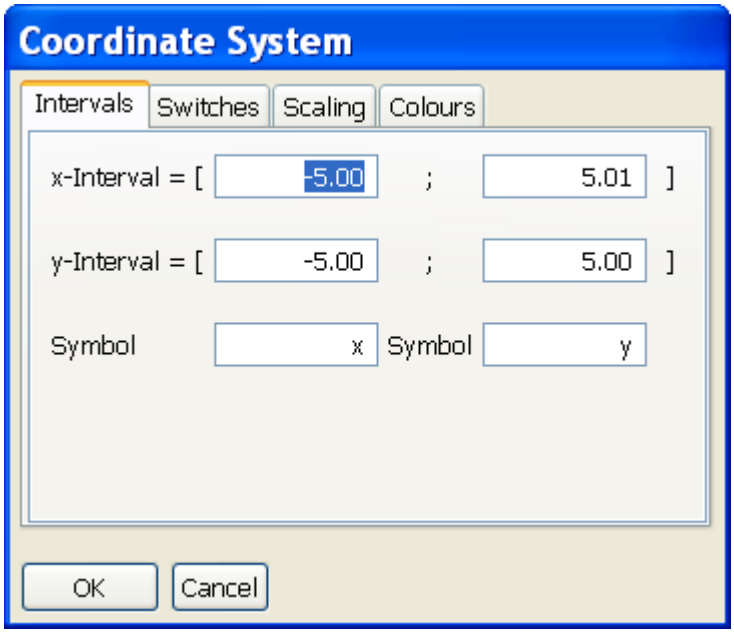
At this point we would like to emphasize that you have to be cautious when deleting objects that are inverse images of transformations. Deleting the these objects will also delete the images!

Menu Coordinate System

This item leads you to all the settings of the coordinate system. There are four tabs with the following titles:

- Intervals
- Switches
- Skaling
- Coulours

Coordinate System – Intervals

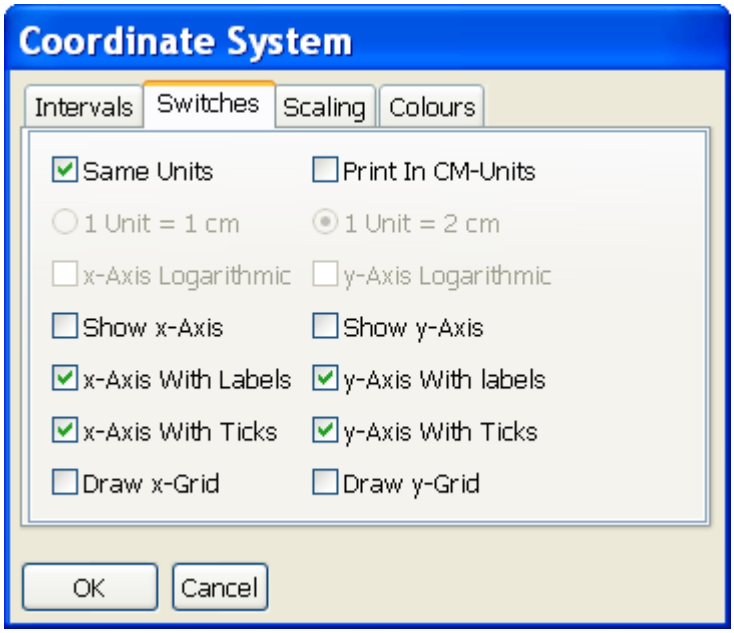


On this tab you specify the borders of the x- and the y-interval. Additionally a

symbol of each axis can be specified.

Since version 6 of the programme you can also determine the borders by drawing a rectangle with the mouse. It is also possible to move the coordinate system by dragging the axes. .

Coordinate System – Switches



There are two special switches:

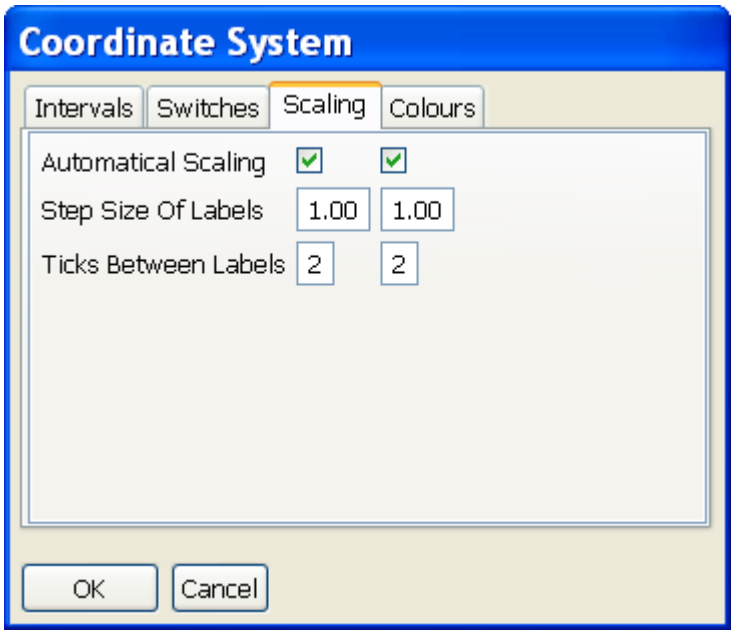
- Identical Units
- Print With cm-Units

Often it is meaningful to have the same units on both axes so that e.g. circles really look like circels. This is accomplished by the first of the special swithces. Activating the second option „Print With cm-Units“ will recalculate the x- and y-intervals on the print page. **This option will possibly change the printed section of the coordinate system considerably**.

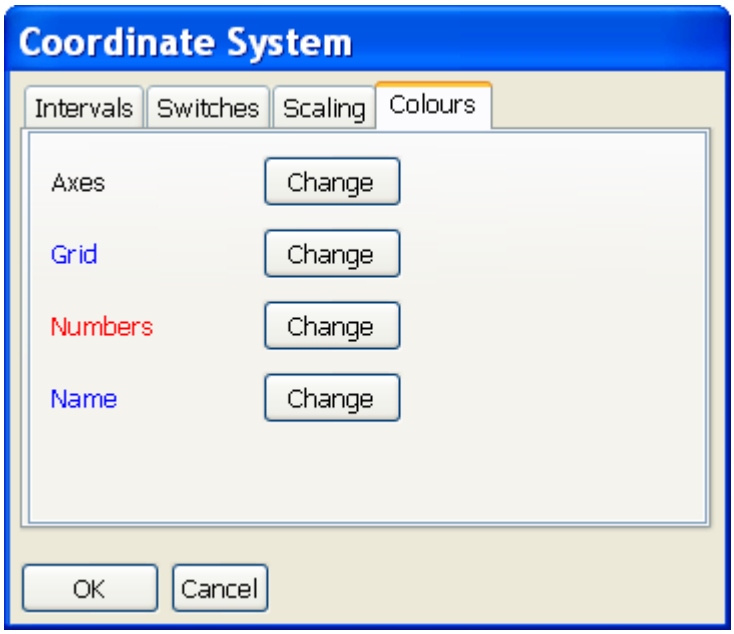
The remaining options allow to switch the scaling ticks, the labels or the drawing of the axes. Additionally a grid can be drawn.

Coordinate System – Scaling

For both axes the scaling and labeling can be done automatically or with special settings. In the last case you have to specify the step size and the number of additional ticks between the labels.



Coordinate System – Colours



•You can provide colours for

- Axes
- Grid
- Labels
- Names

A hint for moving the coordinate system:

If the coordinate system is visible, you can drag the axes to move the whole

scene. It is also possible to use the arrow-keys, which is the only option if the coordinate system is not visible.

Menu Text

You can choose between the following items:

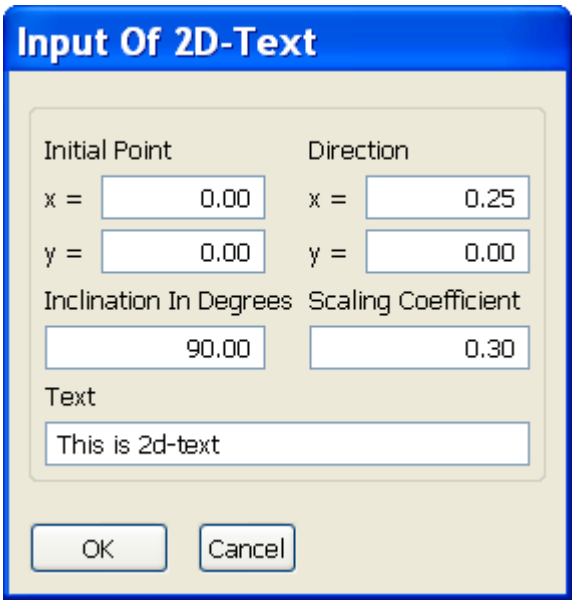
- Title
- Subtitle
- X-Title
- Y-Title
- Text Field
- Directional Text

The first four text inputs will appear at predefined positions:

Title and Subtitle on top of the scene, X-Title at the bottom and Y-Title on the left of the scene. the fastest way to change the titles is double-clicking on them.

A text field is a rectangular region with text. You can move the text field by dragging it with the mouse. To change the text you double-click on it. Further options (like frame or colour) are available by right-clicking on the text field, which brings up the local menu.

The directional text allows for a scalable text including vector symbol and subscript.



The scaling coefficient allows for different text sizes. The programme calculates an appropriate default value, which is a certain percentage of the coordinate section.

Three special control characters can be used together with text:

The symbol **|** applies a vectorial arrow to the following character,

the symbol **_** writes the following character as an index (subscript) and the symbol **^** makes the following character a superscript.

An example:

Input |v_1

The result is $\vec{v}_1$

It is possible to drag the directional text with the mouse or to rotate it while pressing **Ctrl** simultaniously.

Menu Options

These are the current options:

- Decimal digits
- Background white or black
- Search precision
- Number of steps for drawing graphs
- Print with one colour
- Snap mode

Options – Decimal digits

The results of calculations are usually decimal values. This option sets the number of decimal digits that are used for output in the text window on the right.

Options – Background white or black

The default value is a white background, but you can select a black background, if you prefer it. This option is only applicable to the screen output. During printing the programme always selects a white background.

This option will be saved with the graphical data, so that the selected background will be restored when loading the saved file.

Options - Search precision

When searching for zeroes, relative extrema or poles, the programme uses a default precision of 0.0000001. For most purposes this value is reasonable, but there are certain situations where this value should be changed. Anyway, you have to be cautious when when using very small values, because in these cases it could happen that zeroes are not found (because of subtraction errors with very small values).

Options - Number of steps for drawing graphs

When you have to draw graphs with many oscillations, it is adequate to

select a higher value for the number of steps. The maximum value is 2000. Be aware that in this case the drawing is generated less quickly, especially when using families of functions.

Options-Print with one colour

This switch achieves that during printig all colours (except the background colour) will be printed black.

Options - Snap mode

When moving points it is sometimes necessary to end the movement at an already existing point in the scene. Snap mode enables just this. For example, when you want to add one vector to another, you move one vector. If its starting point is near the end-point of the other vector, it „snaps“ exactly to that point, so that both points have the same coordinates.

The snap mode should be **switched off** if it is not needed, because in this mode there are many calculations to be done, especially if there are many objects in the scene. Otherwise this would result in a slower performance.

The Toolbar

Directly below the main menu you find the toolbar which contains buttons with icons for a quick creation of objects. All buttons have a short hint, which shows up if the mouse pointer is positioned over the button for a short time.

If you press the button „point“ and move the mouse pointer in the scene, the cursor shape changes to an small thin cross. Now the programme is in selection mode. The next mouse click determines the point coordinates and switches back to normal mode.

When creating a vector, line segment, straight line or half-line, two mouse-clicks are needed, for triangles three.

A rectangle will be created by selecting the first point and then dragging with the mouse and releasing the button at the diagonally opposite point.

The creation of a circle starts with the centre. Dragging the mouse changes the radius.

Constructing an ellipse is the same procedure as with a rectangle.

Creating a polygon starts with an input dialogue, that queries the number of vertices. After that the adequate number of mouse-clicks has to be performed.

Creating a circular sector needs three mouse clicks. The first is the centre, the second is the starting point on the periphery, the third (counterclockwise) determines the second leg of the sector angle.

All objects (except functions) created via the toolbar are **not anchored**, so that they can be moved with the mouse. During the movement the cursor takes the shape of a hand.

The created objects can be **rotated** with the mouse by pressing the Ctrl-key simultaniously. The cursor then takes the shape of a horizontal double-arrow. Dragging to the left leads to a counterclockwise rotation, dragging to the right to a clockwise rotation.

Some objects (rectangle, circle, circular sector, ellipse, sector of an ellipse) can be scaled with the mouse. This is achieved by having the **Shift**-key pressed when starting the mouse operation (beginning at the object boundary). Dragging the mouse will then change the shape of the object.

Regarding an ellipse, the sensible points for this operation are the end-points of the axes.

The button with the coordinate system switches the axes on or off.

The button **D** starts selection mode where you have to click on two existing objects. The programme will calculate the corresponding distance.

Pressing button **I** starts selection mode, where you are expected to select two objects of which you want to calculate the intersection.

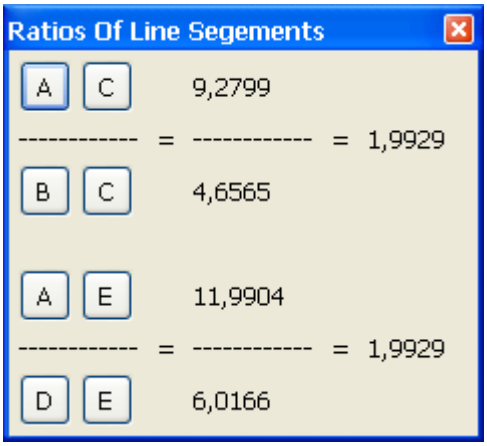
The + and - buttons are zoom buttons.

The remaining buttons are in relation to the data window on the right of the screen.

Ratio Of Line Segments

In the main menu ***New Object*** you can find the entry **window for ratio of line segments**.

If you select this item, a new window like the following one opens in the upper right corner of the canvas area.



You can select points out of the drawing by first pushing one of the buttons P1 to P8 and then selecting an already existing point object. During this operation the cursor takes the shape of an up-arrow.

This way you can specify four distances and two ratios. When points are moved that belong to one of the four line segments, the lengths and the ratios are continously recalculated.

This option is especially helpful when you have intersecting lines that are crossed by two parallel straight lines.

When saving a file, the programme checks whether such a window is present and has valid links to points in the drawing. If so, the links will be reestablished when loading that file.

Local Menus

Invocation

You go to the local menu of an object by right-clicking on the object.

Common Options

These are the menu entries common to all objects:

- Change Data ...
- Name
- Colour
- Linetype
- Transformation by ...
- Show Data
- Delete

Local Menu – Change Data

This option brings up the standard input window for the current object with the actual data. So you can specify the desired values.

Local Menu - Name

This item leads you to an edit box, where you can specify a name for the current object.

If the object is a vector, line segment, halfline, line or a circle, you can work with indices. In these cases you write an underscore followed by the desired subscript.

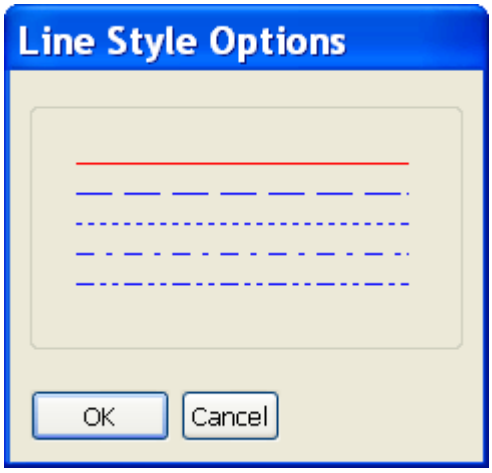
g_1 as input leads to the representation g_1

As to the objects last mentioned, the names can be scaled: Hold Shift pressed while you click on the name, dragging up or down will scale the text (within certain limits).

Local Menu – Colour

This dialogue lets you choose the colour.

Local Menu - Line Style



This dialogue lets you choose the line style.

Local Menu - Transformation by ...

This option offers the opportunity to transform the object with a previously defined affine transformation in the plane. The image will be calculated, added to the list of objects and will be drawn in the scene.

Local Menu – Show Data

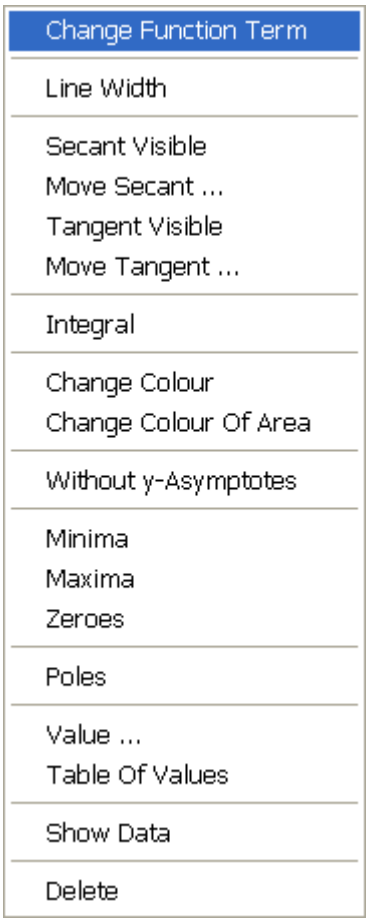
This option shows the actual object data in the text window. It depends on the settings of the text window, whether the contents will be cleared first or not. Right-clicking on the text window opens the local menu, where this can be specified.

Local Menu – Delete

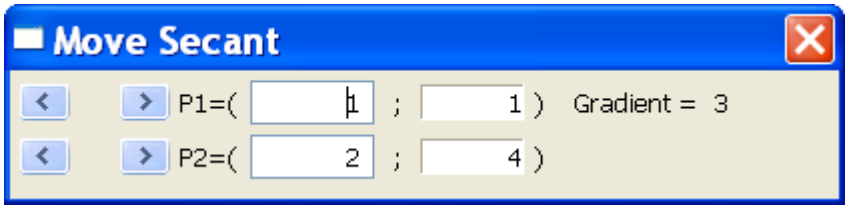
Selecting this item will delete the actual object - after a confirmation. Be cautious: If the object is tied to another, e.g. if the object is the inverse image of a transformation, the image will be deleted, too. Paralles and perpendiculars of the object will not be deleted, but will exist as independent objects. Before the deletion of an object, the whole state of the scene will be saved. In the Edit-Menu there is an item, where you can return to the state before the delete operation (or Ctrl-Z).

The Local Menu f(x)

Functions have a wide variety of options:



Move Secant



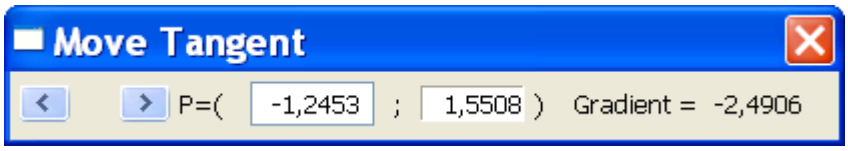
This option activates the window Move Secant. It offers the opportunity to select two points P1 and P2 on the graph, which determine the secant. Pressing the arrow buttons will move the points along the graph, while the secant is adjusted continously.

Secant Visible

This option specifies, whether the secant is visible or not.

Move Tangent / Tangent Visible

Similar to the secant options this item opens a window where you can move a tangent along the graph. The edit field of the x-component can be used to specify the exact position. Pressing ***Enter*** will calculate the y-value and adapt the drawing.



Draw y-Asymptote

As a default asymptotes are drawn, that are parallel to the y-axis. This switch allows an individual setting of this option. This is reasonable e.g. with the hyperbola $f(x) = \frac{1}{x}$, since the asymptote would show the y-axis as a dashed line, which, normally, is not what you want.

Roots

This option calculates the roots of the actual function in the x-interval of the coordinate system. Additionally, you have the opportunity to apply the Newtons method iteratively as an animation.

Maxima, Minima, Poles

These items start the necessary calculations for the selected x-interval of the coordinate system.

Integral

This option calculates the definite integral of the actual function numerically in an interval that has to be determined. You can choose between the following methods:

- Lower Sum
- Upper Sum
- Trapezoidal Method
- Simpson-Method

The number n of subintervals can be specified.

Integral Between

This item lets you calculate the integral between two graphs.

Value

To show certain functional values you select this item. A window with buttons lets you drive along the graph of the function.

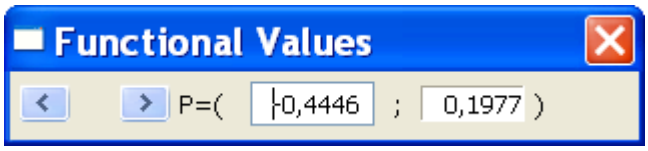
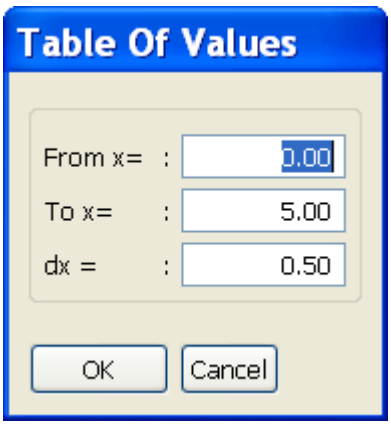
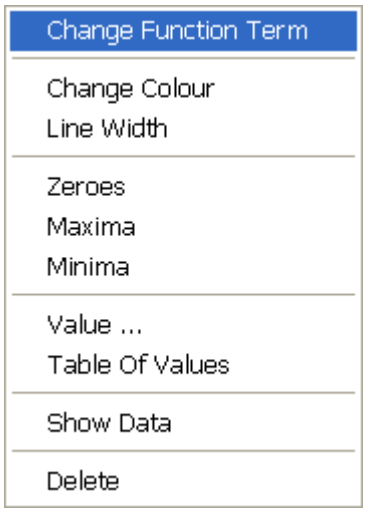


Table Of Values

You can specify initial and final values and the step size for creating the table.



The Local Menu **(x(t);y(t))**



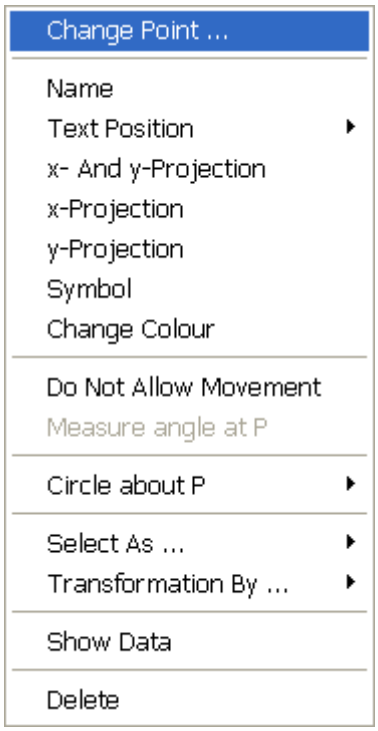
In addition to the common options of all local menus you can select the following:

- Roots
- Maxima
- Minima
- Value
- Table Of Values

The entries are self-explanatory.

The option *Value / Table Of Values* is explained in the section **The Local Menu f(x).**

The Local Menu **Point**



With ***Name*** you can define a label of the point.

Text Position lets you define, where the text appears relative to the point. Eight different positions are offered. The position can also be changed by dragging the text with the mouse. This is true for single points as well as points at the end of a line segment.

Symbol lets you select how to mark the point position.

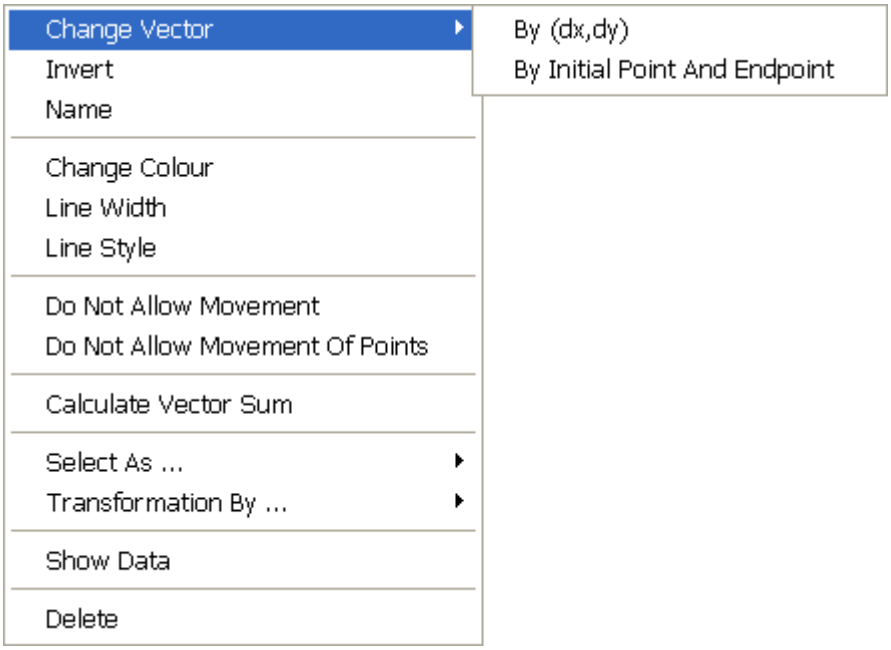
x-Projection and ***y-Projection*** allows to draw projections on the axes of the coordinate system.

Circle Around P With Radius is self-explanatory. The radius has to be determined in an input window. This circle is tied to the point. Moving the point will also move the circle. This option is switched off, if the point alreday is the center of a cirle.

Select As lets you select the actual point as the centre of certain transformations:

- as centre of reflection with respect to this point
- as centre of rotation
- as centre of scaling relative to this point

The Local Menu Vector



You choose between

- Invert
- Line Style
- Calculate Vector Sum
- Select As

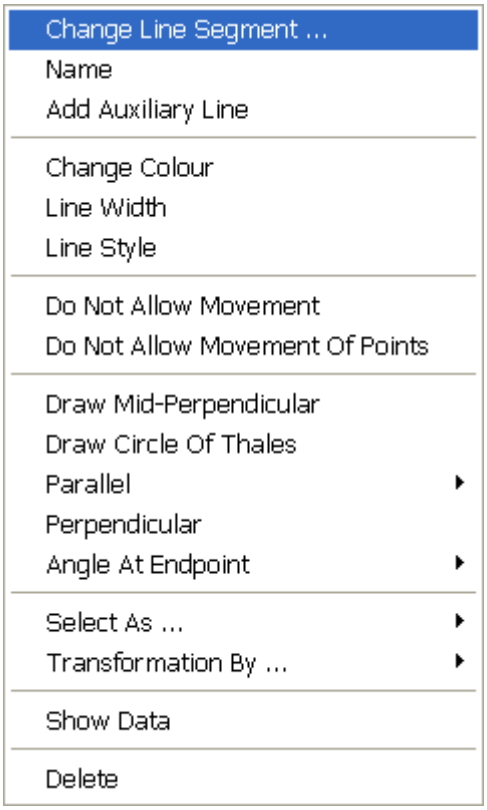
Invert swaps initial and end-point of the vector.

Line Style specifies whether the vector is solid, dashed, dotted or a combination of it.

Calculate Vector Sum checks if the actual vector is summand of a vector chain. If this is the case and the sum has not yet been calculated, this will be achieved now and the resulting vector will be added to the drawing. A subsequent translation of the initial or end-point will adapt the chain accordingly.

Select As translation lets you define a translation in a simple way. If objects are transformed by such a translation and the translational vector changes, the images are adapted automatically.

The Local Menu **Line Segment**



You can select the following:

- Add Auxiliary Line
- Select As
- Mid Perpendicular
- Parallel
- Perpendicular
- Thales Circle
- Angle At Vertex

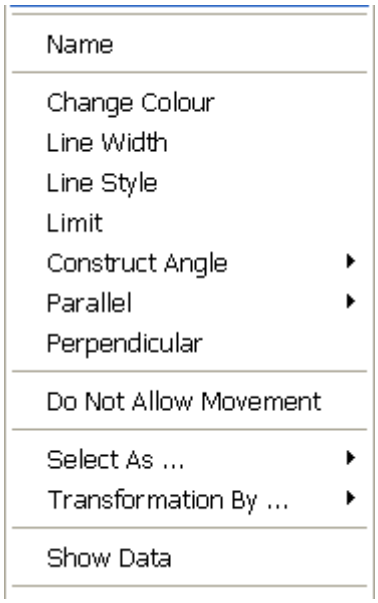
Selecting **Parallel** you will first be prompted for the distance, where the parallel line should appear. After the parallel appears in the drawing it can be moved with the mouse (if it is not anchored).

A **Perpendicular** will be drawn at the click-point.

Select As will define the line defined by the line segment as line of reflection or line of shear. In the case of shear you will be prompted for a coefficient.

Add Auxiliary Line adds a half-line at the end of the line segment, that is next to the click-point.

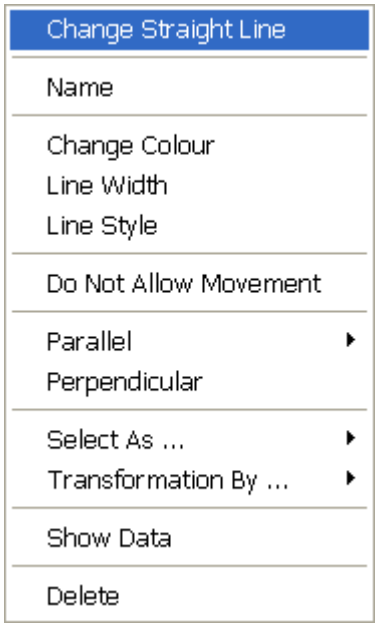
The Local Menu **Half-Line**



The name of a half-line is generated automatically starting with **h** and followed by a numerical index. This can be changed by selecting the item **Name**. The position of this label can be changed by dragging it with the mouse. Changing the font size yields the same procedure as with straight lines.

Especially for clarity of the drawing there is an option ***Limit***. After selecting this item you are expected to click on the half-line, where you want to limit the line. Of course this action can be reversed by selecting **Remove Limit**.

The Local Menu **Straight Line**



The name of a straight line starts with **g** followed by a numerical index.

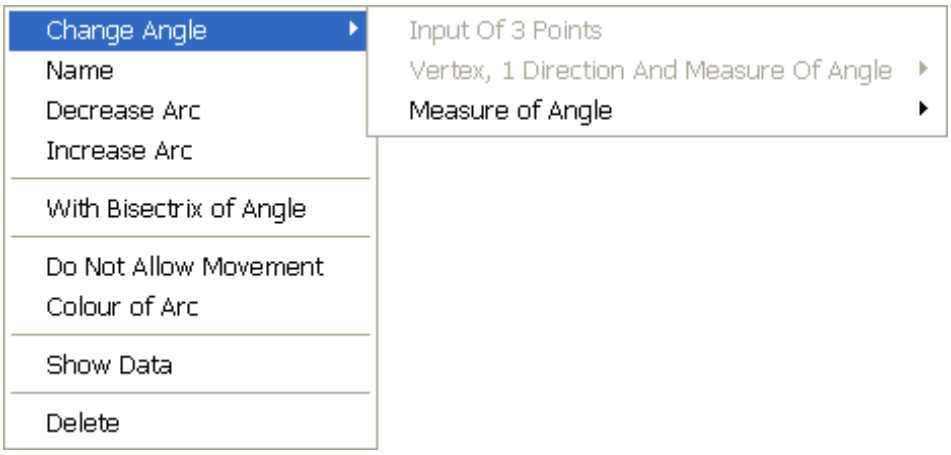
This entry can be changed through the menu item **Name**. The text label can be moved by dragging it with the mouse. To scale the text you start dragging it while having the Shift-key pressed. Upward movement scales it up, downward movement scales it down.

Selecting **Parallel** opens an input window, where you are prompted for the distance. Parallels and perpendiculars can be moved with the mouse, if they are not anchored.

Selecting **Perpendicular** adds a perpendicular at the position of the mouse-click.

The actual straight line can be selected as line of reflection or line of shear by choosing the menu item ***Select As***. This adds an appropriate transformation to the list to transformations. Objects can then be mapped by selecting **Transformation By** in the local menu of the object.

The Local Menu **Angle**



In addition to the usual options you find:

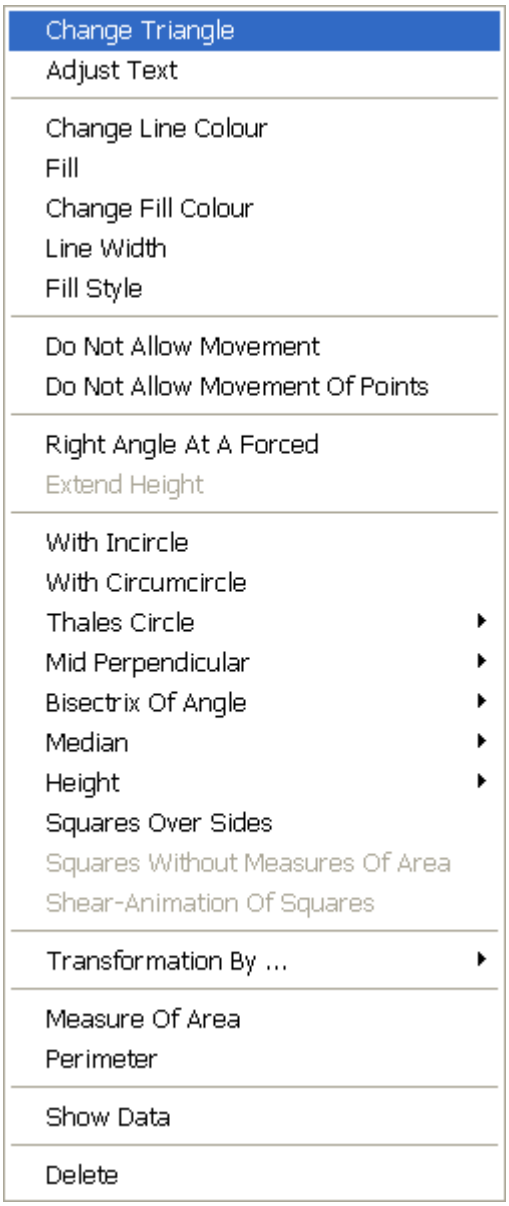
- Bisectrix
- Smaller Arc
- Bigger Arc

The entries are self-explanatory.

It should be added that the inner region of the arc can be used to select the angle (e.g. right-click leads to the local menu).

The Local Menu **Triangle**

These are the options of a triangle:



Adjust Text starts a recalculation of the label positions. This may be necessary if single vertices have been changed.

The triangle can be mapped by affine transformations, that have been defined previously. The operation starts by selecting ***Transformation By***.

It is possible to draw squares over the triangle sides. As default they are filled with certain colours. A right-click on the border of the square brings up the corresponding local menu, where this can be changed.

The squares option is meant to help while exploring the Pythagorean theorem. Only if there is an angle of 90^o the two added areas of the squares are equal to the area of the square over the hypotenuse.

Shear Animation starts an animation, where at first the cathetus squares are sheared to parallelograms and then to rectangles fitting in the hypotenuse sqare.

When triangles are mapped and the option **Draw Squares** is switched on, all images of the transformation will also show squares over the triangle sides.

When you move the triangle or one of its vertices, the images will be adapted accordingly.

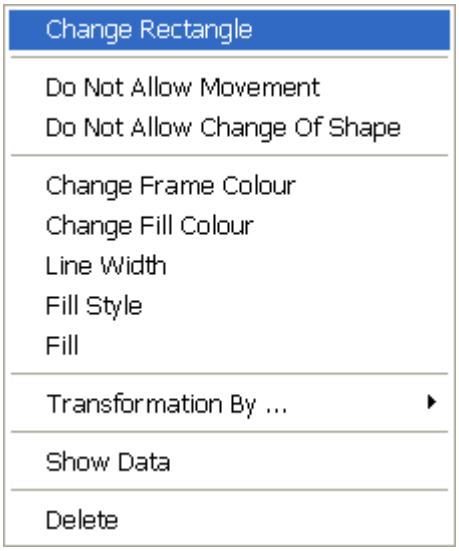
All remaining items are self-explanatory.

We would like to stress that the additional lines like median etc. are not kept in separate object lists but are tied to the triangle. This way they are updated with every change of vertex on one hand, but are not available for calculations of intersections on the other.

Right-clicking on the vertices opens a lot of further options like drawing circles around them. These circles stay tied to the vertices when the triangle is moved or the vertices are changed.

The Local Menu Rectangle

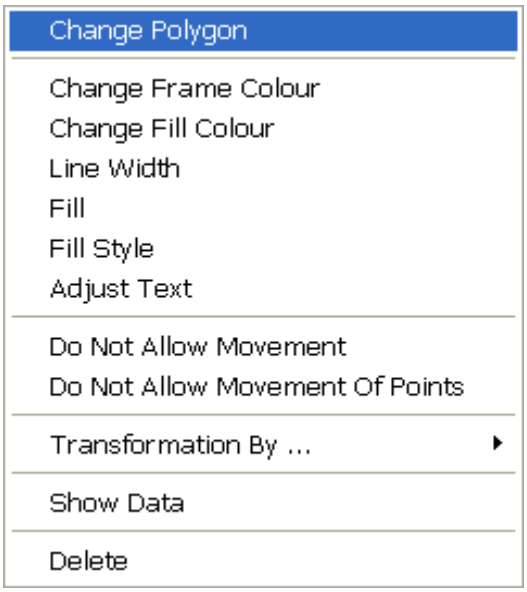
Like all polygons the rectangle offers some filling options.



To deform the rectangle you press **Shift** while dragging the rectangle sides to the desired position.

The Local Menu Polygon

These are the options:



All items are self-explanatory.

The Local Menu Circle

In addition to the standard options you find an item which lets you select only a circular section. After the selection of this item you are expected to click on an initial point and an end-point on the circular line.

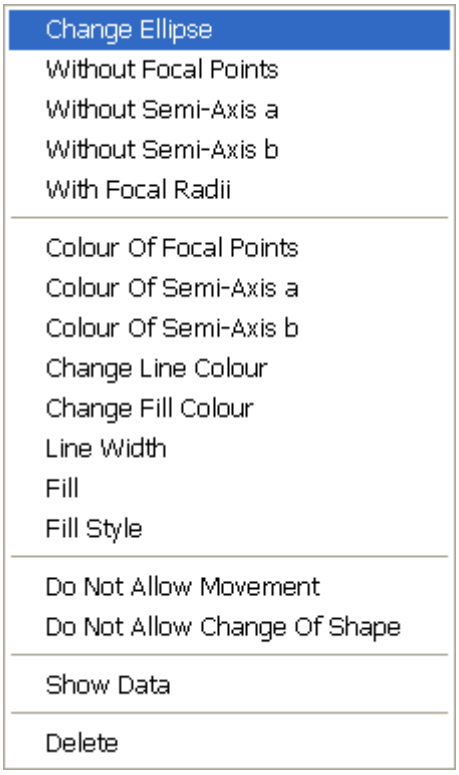
The circle can be scaled by holding the Shift-key pressed while dragging the circular line.

The Local Menu **Circular Sector**

If the switch **Sector Border** is on, the radius lines on both sides of the circular sector are drawn.

You can change the position of the initial and terminal point by dragging these points, holding the Shift-key pressed. The radius can be changed in the same way as with circles.

The Local Menu Ellipse

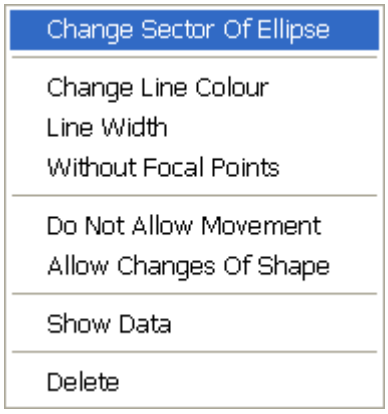


This menu lets you decide to draw focal points, semi-axes and focal radii. If

Focal Radius is switched on, you can move the corresponding point along the peripherals of the ellipse. The radii are adapted during the movement.

In addition you find some filling options.

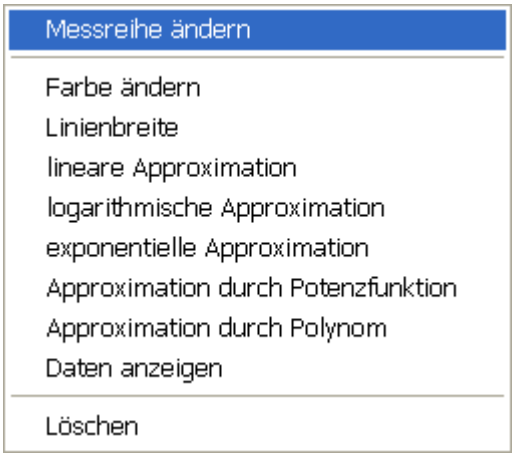
The Local Menu Sector Of An Ellipse



You can decide to draw the focal points.

The Local Menu Series Of Measurement

These are the options:

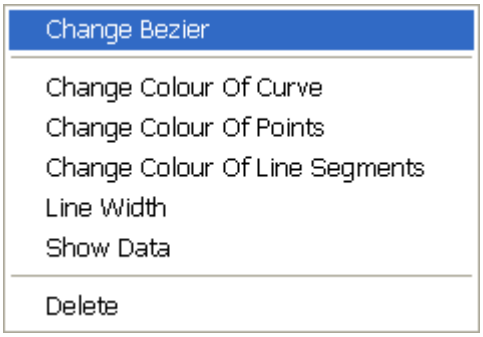


The underlying method of approximation is the method of least squares.

We would like to emphasize that with some approximations only positive values are allowed. If this condition is not true, the approximation will stop.

The Local Menu Bézier

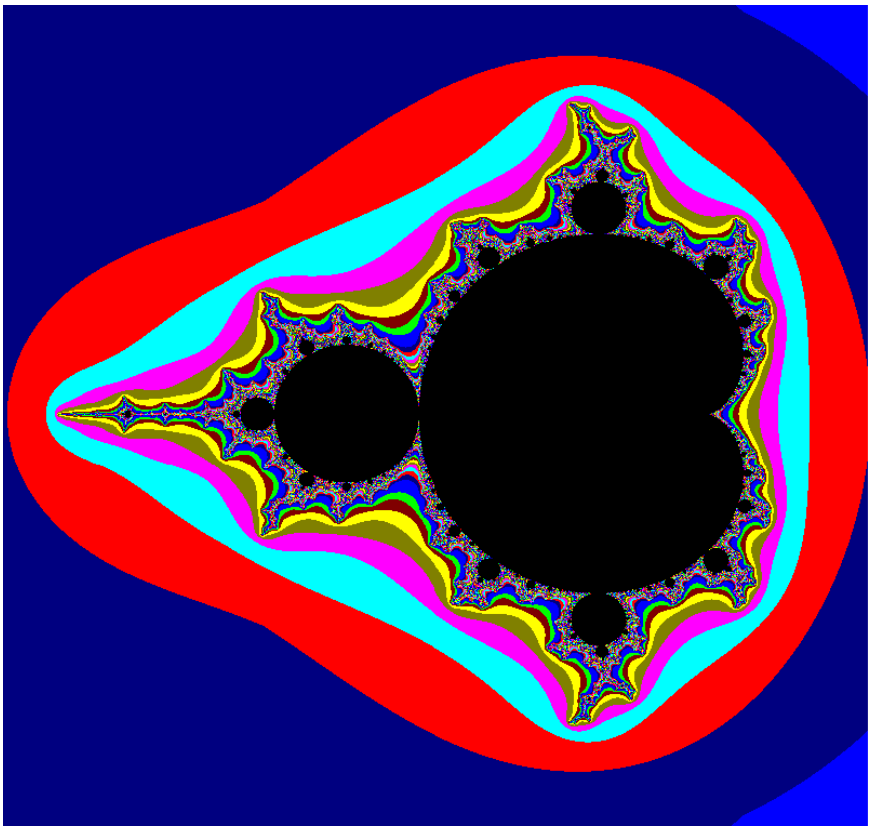
This menu lets you change the colours of the curve, the points and the connecting lines.



Mandelbrot Set

Basics

When selecting **Mandelbrot Set**, the drawing shows a certain rectangular section of the Mandelbrot set.



The coordinate system is interpreted as showing points of the complex number plane. The underlying sequence of numbers is the following:

$$z_{n+1} = z_n^2 + c$$

The starting value (n=0) is always 0, while c is the complex number that is defined by the actual point in the coordinate system.

The whole selected region will be scanned line by line from bottom to top according to the resolution of the screen.

If no divergence of the sequence is detected after a certain number of steps, it is assumed that the sequence is convergent. In this case the point c is coloured with the convergence colour.

If the absolut value of a number in the sequence exceeds a certain value, the point c is coloured in dependence of the number of steps (n).

All points having the same colour lead to the assumption of divergence after the same number of steps in the sequence (z_n).

The resulting image is known as the Mandelbrot set.

To select rectangular regions of the drawing with the mouse leads to interesting results.

Settings Of The Mandelbrot Set

These are the settings, that are accessible through the local menu:

- Number of steps to assumption of convergence
- Show Data

Number of steps

If the section of the coordinate system is very small (e.g. length=0.00001), it is necessary to increase the number of steps in order to show the fine details at the border of the Mandelbrot set. Of course, this setting has a big influence on the computing time.

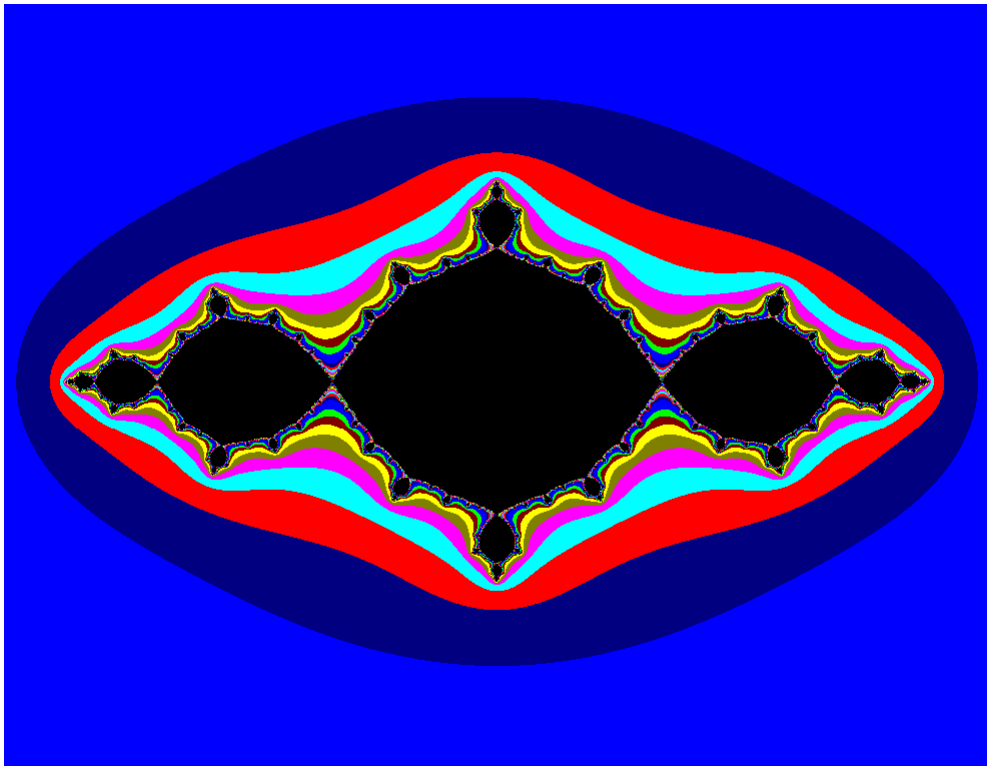
Show Data

This option shows the x- and y-intervals of the selected part of the Mandelbrot set.

Julia Sets

Basics

Selecting **Julia Set** shows a Julia set with a default value of c (see below).



Similar to the Mandelbrot set, the drawing shows certain points that are related to the complex number sequence

$$z_{n+1} = z_n^2 + c$$

The value of c can be arbitrarily chosen and defines the actual drawing. The example drawing is based on an value of c = -1.

The initial value of the sequence (z_o) runs line by line through the selected rectangle (interpreted as part of the complex number plane).

The method of assigning colours is the same as with the Mandelbrot set. If, after a certain number of steps, no divergence is recognized, it is assumed that the sequence is convergent.

If the absolut value of a number in the sequence exceeds a certain value, the initial point is coloured in dependence of the number of steps (n).

Settings Of The Julia Sets

The local menu shows the following options:

- Steps to assumption of convergence

- Change c-value
- Show Data

Steps to convergence

If the section of the coordinate system is very small (e.g. length=0.00001), it is necessary to increase the number of steps in order to show the fine details at the border of the Julia set. Of course, this setting has a big influence on the computing time.

New c-Value

The value of c in the complex sequence defines the structure of the Julia set.

Real and imaginary part of the complex number are prompted.

Show Data

The actual borders of the rectangular Julia section are output.

Licence Information

The Copyright is by:

K.H.Käse-Schulsoftware

Dipl.-Phys. Karl Hans Käse-Kaßbaum

Meisenweg 4

D-25436 Uetersen

WEB: www.kaese-schulsoftware.de

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